# Time varying gravitational constant $G$ via the entropic force

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**Abstract**

If the uncertainty principle applies to the Verlinde entropic idea, it leads to a new term in the Newton's second law of mechanics in the Planck's scale. This curious velocity dependence term inspires a frictional feature of the gravity. In this short letter we address that this new term modifies the effective mass and the Newtonian constant as the time dependence quantities. Thus we must have a running on the value of the effective mass on the particle mass $m$ near the holographic screen and the $G$. This result has a nigh relation with the Dirac hypothesis about the large numbers hypothesis (L.N.H.) [1]. We propose that the corrected entropic terms via Verlinde idea can be brought as a holographic evidence for the authenticity of the Dirac idea.

## I. INTRODUCTION

Recently Verlinde presented a new approach [2] to the nature of the gravitational interaction by a Holographic picture. To illuminate the relation between gravity and thermodynamics, there are some momentous works back to the Padmanabhan which was appeared before Verlinde paper[3, 4]. Verlinde's simple formalism swindles every body to interpret the gravity as an interaction between a screen with coding amount of the information and

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a test particle. It was shown that this form of the the interaction causes a slight change in the entropy. There are many publications on entropic formalism, inspired directly from the work of the Verlinde[5]. The information that was saved on the screen must be modified by some further quantum considerations as the Generalized Uncertainty Principle (GUP) [6] idea for adding some corrections to the Newtonian gravity. One of the most important corrections is the effect of the minimum Plank's length and the modification of the entropic force. As it was shown in [7] this correction which comes from the GUP, is induced as the friction term to the force law, and if we accept that this friction causes motion under a dissipative force then the question is why this term is necessary in the description of the gravity?. Answering to this question is the main goal of us in this short letter. We explicitly show this new dissipative force via the GUP, forces the running to the Newtonian constant $G$ and in a weaker statement the descend of it for future times. The test particle mass also involves similar phenomenon which is the running of the effective mass of the test particle.

## II. PLANCK SCALE CORRECTIONS TO THE GRAVITATIONAL LAW

As it had been shown in Refrence [8] if one introduces Planck scale's minimum length induced directly from the GUP (which is a fundamental counting the cells in the holographic screen ) the quantum makes correct the Newtons second law of motion

$$\overrightarrow{F} = m\overrightarrow{a}\nu - \frac{\overrightarrow{p}}{\tau} \tag{1}$$

where

$$\nu \equiv 1 + \frac{1}{2}(\frac{\lambda}{\delta x})^2 \frac{p}{mc}, \quad \tau^{-1} \equiv \frac{c\lambda}{2(\delta x)^2}, \tag{2}$$

here $\lambda = \frac{\hbar}{mc}$ is the particle's Compton length, $\delta x$ is the displacement of the particle near the screen, and $\overrightarrow{p}$ is the momentum of a particle. For the Kepler i.e. the motion of a particle in a centrifugal force's field, the differential equation (DE) (1) is in the form of the second kind of Abel's equation, class A and as we know that there is no general solution for this DE yet [9]. Only if we neglect the gravitational force and solve the ODE (1) for a free particle we can obtain the following exact result for momentum $p$ in terms of the Lambert function[1]

[1] Consideration of Lambert W function can be traced back to J. Lambert around 1758, and later, it was considered by L. Euler but it was recently established as a special function of mathematics on its own[12].

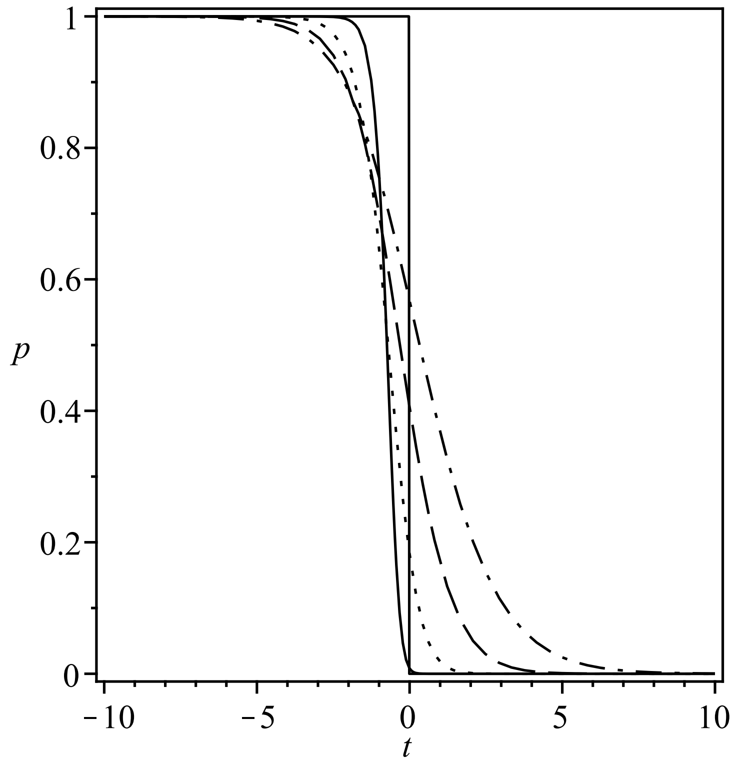


FIG. 1: Variation of the $p$ for a sample of the parameter $\tau$. The value of $\tau$ from down to top is equal to $1, 0.75, 0.5, 0.25, 10^{-4}$.

$$p = e^{-\frac{\tau W(t)-t-c_1}{\tau}}$$

Where $W(t) = LambertW((\frac{\lambda}{\delta x})^2 \frac{e^{\frac{t+c_1}{\tau}}}{2mc})$. The Fig.1 shows the behavior of the rescaled momentum p for some values of the parameter $\tau$.

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The Lambert W function is defined to be the function satisfying $W[z]e^{W[z]} = z$ It is a multivalued function defined in general for z complex and assuming values W[z] complex. If z is real and $z < -1/e$, then W[z] is multivalued complex. If z is real and $-1/e < z < 0$, there are two possible real values of W[z]. The one real value of W[z] is the branch satisfying $W[z] \leq -1$, denoted by $W_0[z]$, and it is called the principal branch of the W function. The other branch is $W[z] \leq -1$ and it is denoted by $W_{-1}[z]$. If z is real and $z \geq 0$, there is a single real value for W[z] which also belongs to the principal branch $W_0[z]$. Special values of the principal branch of the Lambert W function are $W_0[0] = 0$and $W_0[-1/e] = -1$. The Taylor series of $W_0[0]$ about $z = 0$ can be found using the Lagrange inversion theorem and is given by [13]

$$W[z] = -z \sum_{n=0}^{\infty} \frac{(-z)^n (n+1)^n}{n!}$$

The ratio test establishes that this series converges if $\mid z \mid < 1/e$.

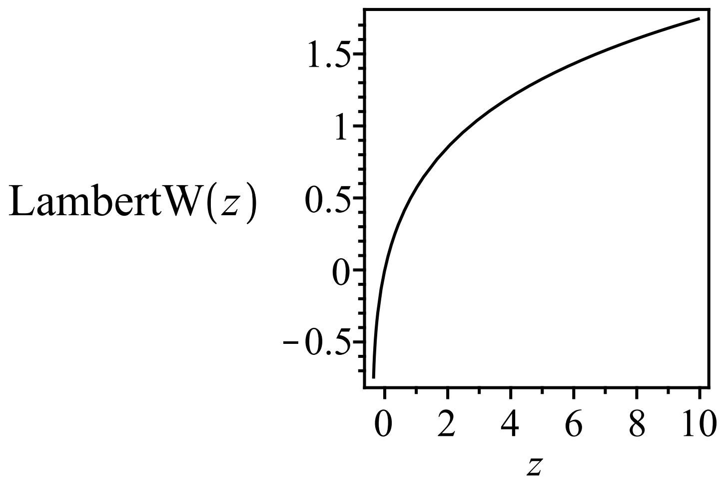


FIG. 2: Variation of the $LambertW(z)$ .

For small momentum by transferring $p$ in the regime of non relativistic test particle near holographic screen $\frac{p}{mc} << 1$ we can neglect from the second term of the $\nu$ in (2) and take it as unity i.e. $\nu \simeq 1$. We mention here that a relativistic gravitational field can not be considered and as we know that which further detailed that requires more general relativity theory. In the Verlinde original proposal for gravity he neglected the curve space effects at the first step. Thus we set $\nu = 1$.

## III. THE HAMILTONIAN OF A TEST PARTICLE

The simple time dependence Hamiltonian for the system (1) in a conservative central force field

$$V(r) = -\frac{K}{r} \tag{3}$$

where $K = GMm$ is read from the comparing (1) with a one dimensional dissipative simple harmonic oscillator in classical mechanics [10]. We find the Hamiltonian describing the test particle of mass m moving under the influence of the central potential to be

$$H = \frac{\overrightarrow{p}.\overrightarrow{p}}{2m} e^{\frac{t}{\tau}} + e^{-\frac{t}{\tau}} V(r) \tag{4}$$

our Hamiltonian is rotational invariant exactly as the $\kappa$ spacetime like in a noncommutative version of the Kepler's problem [11]. Also we note that the modification for the kinetic term

can be captured by a suitable definition of the mass as an effective mass $m_{eff}$

$$m_{eff} = me^{\frac{-t}{\tau}} \tag{5}$$

Here, we also note that replacing m with $m_{eff}$ in Lagrangian does not affect the rotational invariance of the system described by above Hamiltonian. Comparing (4) with (5) testifies that the superficial constant of the $k$ must have the following running:

$$K = GMme^{\frac{-t}{\tau}} \tag{6}$$

Note that (6) recovers the classical results in limit $\hbar \to 0, \tau \to \infty$ but queer in a curious running scheme. From 5 we conclude that in the entropic scenario, the screen does not sense the usual classical mass $m$ but it senses an effective time dependence mass which it decays with time in GUP regime. This feature shows that in general, there are more ambiguities for applying the Verlinde idea based on the usual equilibrium statistical mechanics. In the next sections we show that the modification of the usual gravitational constant is a good evidence for proposing a new Dirac large numbers hypothesis. But before doing it, we review Dirac's idea about the existence of dimensionless large numbers in nature.

## IV. DIRAC LARGE NUMBERS HYPOTHESIS AND ENTROPIC RUNNING OF THE $G$

In this section first we review the Dirac's theory and then we will show some similarities of our result with Dirac's hypothesis.

### A. Dirac Large Numbers Hypothesis

There are three dimensionless numbers in the nature which can be constructed from the atomic and cosmological datas:

1-The ratio of the electric to the gravitational force between an electron and a proton $7\times 10^{39}$

2-The age of the Universe t, expressed in terms of a unit of time provided by atomic constants $\frac{e^2}{m_e c^3}$

and finally

3- *The mass of that part of the Universe that is receding from us with a velocity* $v < c/2$

*expressed in units of the proton mass of the order* $10^{78}$
Dirac large number hypothesis in it's orthodoxies form Germaned by himself says that "*..these numbers are related by equations in which the coefficients are close to unity*". Since the number in (2) varies with the age of the Universe, the L.N.H. requires that the other numbers must also vary, namely

$$\frac{e^2}{Gm_e m_p} \propto t \tag{7}$$

or

$$N \propto t^2 \tag{8}$$

$$G \propto t^{-1} \tag{9}$$

There are two interpretations for the above relation both discussed by Dirac and the only one which was acceptable by himself as *One can reconcile the relation (9) with conservation of mass by assuming that the velocity of recession of a galaxy is continually decreasing, so that more and more galaxies are continually appearing with velocity of recession* $< c/2$.
This is the picture which was adopted in his first paper on the subject [12]. There are a serious problem between (9) and GR: the Einstein's theory requires G to be constant. As was noted by Dirac's theory this inconsistency might be solved if"*.. we assume that the Einstein theory is valid in a different system of units from those provided by the atomic constants."*

### B. Entropic corrections as a new L.N.H.

First we write (6) as the following form:

$$G = G_N e^{\frac{-t}{\tau}} \tag{10}$$

Where the $G_N$ is the usual Newtonian gravitational constant. The quantum Planck's corrections from GUP are significance only at Planck's time scales for example in GUT epoch. Thus essentially the time coordinate $t$ is very small in comparison with the new scale $\tau$. We can expand the exponential term in (10) in powers of the $t/\tau$:

$$G = G_N\left(1 - \frac{t}{\tau} + \frac{t^2}{2\tau^2} + ..\right) \tag{11}$$

Thus for enough small times comparable with $\tau$ the linear term dominates,thus the new L.N.h. is

$$N \propto (1-(\frac{t}{\tau})^2) \tag{12}$$

$$G \propto (1-\frac{t}{\tau}) \tag{13}$$

## V. COMPARING THE MODEL WITH OBSERVATIONAL CONSTRAINTS

From observational view, as it was shown [13], there is a secular decrease of the gravitational constant $G(t)$ . This time rate of change is of order $\frac{\dot{G}}{G}=(-5.9\pm 4.4)\times 10^{-14}yr^{-1}$ which it has been obtained from planetary data analysis. It is convient that we compare (10) with observational bounds. Remembering to mind that, the time scale of our model is $\tau=\frac{2(\delta x)^2}{c\lambda}$. It's physically reasonable , we compare this time scale in regime that the particle's replacement $\delta x$ be of order of the Compton's wave length of the test particle. This means that $\delta x\sim O(\lambda)$. For example , we take $\delta x=\lambda$ , then we lead to $\tau=\frac{2\lambda}{c}$. It is possible to show that in this case $\frac{\dot{G}}{G}=-\frac{2mc^2}{h}$. We take the test particle as an electron with mass $m=0.511Mev/c^2$, then $\frac{\dot{G}}{G}=O(10^{-14}yr^{-1})$. Thus, our theoretical result is compatible with the observational reports, if we take the element of the particle's displacement of order the Compton wave length of the electron, as a test particle.

## VI. SUMMARY

Briefly we show that the new corrections from GUP were imposed on the entropic nature of the gravity,by possessing a running value for the $G$. This simple derivation shows that there is a ticklish relation between the entropic corrections and the GUP and also on which happened in a $\kappa$ space analysis of the Kepler's problem in non commutative spacetimes. It is a good idea to investigate the relation between non commutative spacetimes and the entropic origin of the gravity. Also there is a L.N.h. like relation for this theory. Thus there must exist a tight relation between Holographic description of the Gravity, non commutativity, L.N.h. Thus any unification of the Gravity and the Quantum physics will be hanker between these features.

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